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RM-3369-PR
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SUPPORT REQUIREMENTS FOR OPPORTUNISTIC REPLACEMENT AND INSPECTION POLICIES

J. J. McCall

PREPARED FOR:

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The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum is a continuation of RAND research in the area of preferred maintenance policies. Previous RAND work on optimal replacement and inspection policies has been concerned primarily with demonstrating the optimality of policies possessing an opportunistic structure, developing methods for computing the best values of the critical numbers associated with various policies, and applying these policies to Air Force maintenance problems. This research has been reported in a series of Memoranda which includes: M. Kamins, Determining Checkout Intervals for Systems Subject to Random Failures, Memorandum RM-2578, June, 1960; D. W. Jorgenson and J. J. McCall, Optimal Replacement Policies for a Ballistic Missile, Memorandum RM-3101-PR, May, 1962; and R. Radner and D. W. Jorgenson, Opportunistic Replacement of a Single Part in the Presence of Several Monitored Parts, Memorandum RM-3057-PR, November, 1962.

The purpose of this Memorandum is to provide a method that inventory and maintenance managers may use to predict the effects of different maintenance policies. As an example the opportunistic replacement policy described in RM-3101-PR is applied to a hypothetical ballistic missile.

The results of this Memorandum will be included in a forthcoming Report which will summarize RAND research on opportunistic maintenance policies.

SUMMARY

This Memorandum calculates some of the support requirements for several opportunistic replacement and inspection policies. Support requirements like these are quite important since they constitute the basic and relatively unexplored link between maintenance policies and inventory policies.

An opportunistic replacement policy makes the replacement of a single uninspected part conditional on the state (good or failed) of one or more continuously inspected (monitored) parts. An opportunistic inspection policy makes the inspection of a non-monitored part conditional on the state (good or failed) of a monitored part.

Some of the important support requirements of these policies examined in this Memorandum are: the expected number (per unit time) of opportunistic (joint) replacements of the uninspected part and one of the monitored parts; the expected number (per unit time) of planned replacements of the uninspected part; the probability of a certain number of failures of a monitored part in a specified interval of time; the expected number (per unit time) of opportunistic inspections -- inspections of the non-monitored part which are triggered by failures of the monitored part; and the expected number (per unit time) of planned inspections of the non-monitored part. As an example the opportunistic replacement policy is applied to the rocket engines of a hypothetical ballistic missile. Several support requirements are then computed and the sensitivity of these support requirements to changes in the rocket engine failure rate is exhibited. This illustrative analysis indicates that both the expected number

(per unit time) of opportunistic (joint) replacements of the rocket engines and re-entry vehicle <u>and</u> the expected number (per unit time) of replacements of the rocket engines due to mandatory replacement are highly sensitive to changes in the rocket engine failure rate. On the other hand, the expected number (per unit time) of opportunistic (joint) replacements of the rocket engines and the guidance and control system is relatively unaffected by changes in the engine failure rate.

ACKNOWLEDGMENTS

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CONTENTS

I. INTRODUCTION

Previous RAND work on optimal replacement and inspection policies has been primarily concerned with demonstrating the optimality of policies possessing an opportunistic structure,[1][2] developing methods for computing the best values of the critical numbers associated with various policies,[1-6] and applying these policies to Air Force maintenance problems.[3, 4, 6-8] The purpose of this Memorandum is to investigate the support requirements for the opportunistic replacement and inspection policies.

An opportunistic replacement policy makes the replacement of a single uninspected part conditional on the state (good or failed) of one or more continuously inspected (monitored) parts. The policy is effective whenever it is less costly to replace the uninspected part and one of the monitored parts together rather than replacing each separately. When such economies of scale are present, three different replacement actions will be observed: replacement of the uninspected part by itself (planned replacement), the joint (opportunistic) replacement of one of the monitored parts and the uninspected part, and replacement of one of the monitored parts by itself.

The monitored parts are assumed to fail exponentially and are replaced only at the time of failure. The uninspected part fails according to an arbitrary probability distribution. All replacements are assumed to be instantaneous. Some of the important support requirements of this policy are: the expected number (per unit time) of opportunistic replacements of the uninspected part and one of the monitored parts, the expected number (per unit time) of planned

replacements of the uninspected part, the expected number (per unit time) of replacements of each of the monitored parts, and the probability of at least m failures of a monitored part in the interval [0,t].

An opportunistic inspection policy makes the inspection of a non-monitored part conditional on the state of a monitored one. The policy is effective whenever it is less costly to replace both parts together rather than replacing each separately.* Both parts are assumed to fail exponentially. Some of the important support requirements (each measured per unit time) of this policy are: the expected number of opportunistic inspections -- inspections of the unmonitored part which are triggered by failures of the monitored part; the expected number of planned inspections -- inspections of the non-monitored part due to the passage of a critical amount of time; the expected number of opportunistic replacements; and the expected number of planned replacements.

Support requirements for such maintenance policies are precisely the information needed to establish a suitable supply policy. Indeed, support requirements of this kind constitute the basic and relatively unexplored link between maintenance policies and inventory policies.

Support requirements are examined in Sec. II for a simple opportunistic replacement policy -- the optimal replacement of an uninspected part in the presence of a single inspected part. Similar support requirements are derived in Sec. III for a simple opportunistic inspection policy. Section IV investigates support requirements for the general opportunistic replacement policy -- the optimal

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*The exact values of the policy parameters will also depend on the cost of inspection.

replacement policy for an uninspected part in the presence of a finite number of inspected parts. Section V presents a numerical example and Sec. VI offers some concluding remarks.

II. SUPPORT REQUIREMENTS FOR THE (n, N) OPPORTUNISTIC REPLACEMENT POLICY

PRELIMINARIES

The optimal replacement of a single uninspected part, part 0, in the presence of a single inspected part, part 1, gives rise to an opportunistic replacement policy characterized by two critical numbers, n and N, such that:

(1) If part 1 fails in the interval $0 \le t < n$, replace part 1 by itself;

(2) If part 1 fails in the interval $n \le t < N$, replace part 0 and part 1 together;

(3) If part 0 has not been replaced in the interval $0 \le t \le N$, replace part 0 by itself at $t = N$;

where t is the age (since last replacement) of part 0.*

The time-to-failure of part 0 is a random variable with an arbitrary probability distribution. Part 1 is assumed to fail exponentially with failure rate λ_1. This description implies that the time V until the first replacement of part 0 is a random variable with a probability density function:

$$f(V) = \begin{cases} \lambda_1 e^{-\lambda_1(V-n)}, & n \le V < N \\ e^{-\lambda_1(N-n)}, & V = N \\ 0, & \text{elsewhere.} \end{cases} \tag{1}$$

The following support requirements will be calculated:

$\dfrac{E(R_1)}{t}$ = expected number (per unit time) of replacements of part 1;

---

*See Ref. 1.

$p_1(m, t)$ = probability of at least m failures of part 1 in the interval $[0, t]$;

$\frac{E(R_{01})}{t}$ = expected number (per unit time) of opportunistic (joint) replacements of part 0 and part 1;

$\frac{E(R_{00})}{t}$ = expected number (per unit time) of planned replacements of part 0 (replacement of part 0 by itself);

$\frac{E(R_{0.})}{t}$ = expected number (per unit time) of replacements of part 0.

Notice that:

$$\frac{E(R_{0.})}{t} = \frac{E(R_{01})}{t} + \frac{E(R_{00})}{t} . \tag{2}$$

REPLACEMENTS OF THE INSPECTED PART

The assumption of exponentiality implies that $\frac{E(R_1)}{t}$, the expected number (per unit time) of replacements of part 1 is given by:

$$\frac{E(R_1)}{t} = \lambda_1 . \tag{3}$$

By the same assumption $P_1(m, t)$, the probability of at least m replacements in the interval $[0, t]$ can be expressed as:

$$P_1(m, t) = \sum_{j=m}^{\infty} e^{-\lambda_1 t} \frac{(\lambda_1 t)^j}{j!} .^{*} \tag{4}$$

REPLACEMENTS OF THE UNINSPECTED PART

The replacement of part 0 is clearly a renewal process and hence all of the results of renewal theory are at our command. However, only the simplest result is required -- the elementary renewal theorem.[10][11] By means of this theorem it can be shown that

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*See p. 130 of Ref. 9 or pp. 401-402 of Ref. 10.

$\frac{E(R_{0.})}{t}$, the expected number (per unit time) of replacements of part 0 is asymptotically equal to the reciprocal of the expected value of V, the time to the first replacement of part 0. That is,

$$\lim_{t\to\infty} \frac{E(R_{0.})}{t} = E(V)^{-1} \tag{5}$$

where

$$E(V) = n + \frac{1}{\lambda_1}\left(1-e^{-\lambda_1(N-n)}\right). \tag{6}$$

Hence, for large values of t,

$$\frac{E(R_{0.})}{t} \cong \frac{1}{n+\frac{1}{\lambda_1}\left(1-e^{-\lambda_1(N-n)}\right)} \;.^{*} \tag{7}$$

By the law of large numbers, the fraction of these replacements of part 0 which are opportunistic is, in the long run, approximately equal to the probability that part 1 will fail in the interval [0, N-n] which is:

$$Pb(X \le N-n) = 1 - e^{-\lambda_1(N-n)}. \tag{8}$$

It follows that for large values of t, $\frac{E(R_{01})}{t}$, the expected number (per unit time) of opportunistic replacements of part 0 is:

---

*This is the mean of the asymptotic normal distribution with variance: $\lim_{t\to\infty} \frac{Var\ (R_{0.})}{t} = \frac{Var^2(V)}{E^3(V)}$. See Ref. 11.

(9) $$\frac{E(R_{01})}{t} \simeq \frac{1-e^{-\lambda_1(N-n)}}{n+\frac{1}{\lambda_1}(1-e^{-\lambda_1(N-n)})}\ ,^{*}$$

and $\frac{E(R_{00})}{t}$ the expected number (per unit time) of planned replacements of part 0 is:

(10) $$\frac{E(R_{01})}{t} \simeq \frac{e^{-\lambda_1(N-n)}}{n+\frac{1}{\lambda_1}(1-e^{-\lambda_1(N-n)})}\ .$$

The calculation of the expected number of replacements of part 0 (planned or opportunistic) in the finite interval [0, t] and the corresponding probability distributions involve the inversion of non-standard La Place transforms. For example, the La Place transform of $E_t(R_{0.})$, the expected number of replacements of part 0 in the interval [0, t] is given by:

(11) $$L(s) = \frac{e^{-ns}(1+se^{-((N-n)(s+1))})}{s+1-e^{-ns}(1+se^{-((N-n)(s+1))})}\ .$$

Stranger transforms appear when one attempts to calculate the probability of at least m replacements of part 0 in the interval [0, t].**

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*See Appendix for a more rigorous derivation.

**However, these support requirements have been calculated for n = 0. See Ref. 9.

III. SUPPORT REQUIREMENTS FOR THE (n, N) OPPORTUNISTIC INSPECTION POLICY

PRELIMINARIES

The optimal inspection of a single part, part 0, in the presence of a monitored part, part 1, also gives rise to an opportunistic policy characterized by two critical numbers, n and N, such that:

(1) If part 1 fails in the interval, $0 \le t < n$, replace part 1 by itself.

(2) If part 1 fails in the interval, $n \le t < N$, inspect part 0; if part 0 has not failed replace part 1 by itself; if part 0 has failed replace both parts.

(3) If part 0 has not been inspected in the interval $0 \le t \le N$ inspect part 0 at $t = N$ and replace it if it has failed.* The age (since last inspection) of part 0 is denoted by t.

Both parts are assumed to fail exponentially with failure rates λ_0 and λ_1, respectively. This description implies that V, the time until the first inspection of part 0, is a random variable with a probability density function:

$$
f(V) = \begin{cases} \lambda_1 e^{-\lambda_1(V-n)}, & n \le V < N \\ e^{-\lambda_1(N-n)}, & V = N \\ 0, & \text{elsewhere} . \end{cases} \tag{12}
$$

The probability that an inspection will not induce a failure in a good part is denoted by σ.

The following support requirements will be calculated:

$\frac{E(I_{0.})}{t}$ = expected number (per unit time) of inspections of part 0.

*See Ref. 1.

$\frac{E(I_{01})}{t}$ = expected number (per unit time) of opportunistic inspections of part 0, that is, inspections which have been provoked by a failure of part 1.

$\frac{E(I_{00})}{t}$ = expected number (per unit time) of planned inspections of part 0, that is, inspections which occur because part 0 has reached the critical age, N.

$\frac{E(R_{01})}{t}$ = expected number (per unit time) of opportunistic replacements of part 0.

$\frac{E(R_{00})}{t}$ = expected number (per unit time) of planned replacements of part 0.

INSPECTIONS OF THE NON-MONITORED PART

The non-monitored part, part 0, is assumed to fail exponentially, and is replaced whenever a failure is observed. Consequently, inspections of part 0 constitute a renewal process and the calculation of these support requirements entails essentially the same argument as in Sec. II. Hence, for large values of t,

$$(13) \qquad \frac{E(I_{0.})}{t} \cong \frac{1}{n + \frac{1}{\lambda_1}\left(1-e^{-\lambda_1(N-n)}\right)},$$

$$(14) \qquad \frac{E(I_{01})}{t} \cong \frac{1 - e^{-\lambda_1(N-n)}}{n + \frac{1}{\lambda_1}\left(1-e^{-\lambda_1(N-n)}\right)},$$

and

$$(15) \qquad \frac{E(I_{00})}{t} \simeq \frac{1 - e^{-\lambda_1(N-n)}}{n + \frac{1}{\lambda_1}\left(1-e^{-\lambda_1(N-n)}\right)},$$

REPLACEMENTS OF THE NON-MONITORED PART

In the long run P, the proportion of opportunistic inspections which reveal a failure is:

$$P = 1 - \sigma\pi$$

where σ is the probability that an inspection will not cause a failure and π is the expected value of the reliability function of part 0 in the interval, $n \leq t \leq N$. Accordingly, $\frac{E(R_{01})}{t}$, the expected number of opportunistic replacements (per unit time) is:

$$(16) \qquad \frac{E(R_{01})}{t} \simeq (1-\sigma\pi)\,\frac{E(I_{01})}{t},$$

where

$$(17) \qquad \pi = \frac{\lambda_1 e^{-\lambda_0 n}\left(1-e^{-(\lambda_0+\lambda_1)(N-n)}\right)}{(\lambda_0+\lambda_1)\left(1-e^{-\lambda_1(N-n)}\right)}.$$

Similarly, $\frac{E(R_{00})}{t}$ the expected number of planned replacements (per unit time) is:

$$(18) \qquad \frac{E(R_{00})}{t} \simeq \left(1 - \sigma e^{-\lambda_0 N}\right)\frac{E(I_{00})}{t}.$$

IV. SUPPORT REQUIREMENTS FOR THE GENERAL OPPORTUNISTIC REPLACEMENT POLICY

PRELIMINARIES

The optimal replacement of a single uninspected part, part 0, in the presence of a finite number of inspected parts, part 1, part 2, ..., part I, gives rise to an opportunistic policy characterized by the following decision rule with I+1 critical numbers, n_1, n_2, ..., n_I, N:

(1) If part i, i = 1, 2, ..., I, fails in the interval, $0 \leq t \leq n_i$, replace part i only;

(2) If part i fails in the interval, $n_i \leq t < N$, replace part i and part 0 together;

(3) If part 0 has not been replaced in the interval $0 \leq t \leq N$, replace part 0 by itself at t = N,

where t is the age (since last replacement) of part 0.*

Each of the inspected parts is assumed to fail exponentially with failure rate λ_i , i = 1, 2, ..., I. The time-to-failure of the uninspected part is a random variable with an arbitrary probability density function. This description implies that V, the time until the first replacement of part 0 is a random variable with probability density function:

---

*See Ref. 2.

$$
(19) \qquad p(V) = \begin{cases} 0, & 0 \le V < n_1 \\ \lambda_1 e^{-\lambda_1 (V-n_1)}, & n_1 \le V < n_2 \\ \vdots & \\ \left(\sum_{j=1}^{i} \lambda_j \right) \left(e^{-\sum_{j=1}^{i} \lambda_j (V-n_j)} \right), & n_i \le V < n_{i+1} \\ \vdots & \\ e^{-\sum_{j=1}^{I} \lambda_j (N-n_j)}, & V = N \\ 0 & \text{elsewhere,} \end{cases}
$$

where $0 \le n_1 \le n_2 \le, \ldots, \le n_{I-1} \le n_I \le N$.

The following support requirements will be investigated:

$\frac{E(R_i)}{t}$ = expected number (per unit time) of replacements of part i, $i = 1, 2, \ldots, I$.

$P_i(m, t)$ = probability of at least m failures of part i in the interval $[0, t]$.

$\frac{E(R_{0i})}{t}$ = expected number (per unit time) of opportunistic (joint) replacements of part 0 and part i.

$\frac{E(R_{00})}{t}$ = expected number (per unit time) of planned replacements of part 0 (replacement of part 0 by itself).

$\frac{E(R_{0.})}{t}$ = expected number (per unit time) of replacements (planned plus opportunistic) of part 0.

Notice that this last expected value can be expressed:

$$
(20) \qquad \frac{E(R_{0.})}{t} = \frac{1}{t} \sum_{i=0}^{I} E(R_{0i}) .
$$

REPLACEMENT OF THE INSPECTED PARTS

The time-to-failure for each of the inspected parts is a random variable with an exponential probability density function. This implies that $\frac{E(R_i)}{t}$, the expected number (per unit time) of replacements of part i, is given by:

$$\frac{E(R_i)}{t} = \lambda_i, \; i = 1, 2, \ldots, I. \tag{21}$$

The assumption of exponentiality also implies that $P_i(m, t)$, the probability of at least m replacements of part i in the interval [0, t], is given by:

$$P_i(m, t) = \sum_{j=m}^{\infty} e^{-\lambda_i t} \frac{(\lambda_i t)^j}{j!}^{*}. \tag{22}$$

REPLACEMENTS OF THE UNINSPECTED PART

The calculation of the support requirements for part 0 is easily accomplished by means of the elementary renewal theorem.(11) Using this theorem it can be shown that $\frac{E(R_{0.})}{t}$, the expected number (per unit time) of replacements of part 0 is asymptotically equal to the reciprocal of the expected value of V, the time to the first replacement of part 0. Symbolically,

$$\lim_{t\to\infty} \frac{E(R_{0.})}{t} = \mu^{-1} \tag{23}$$

where

---

*See p. 130 of Ref. 9, and pp. 401-402 of Ref. 10.

(24) $$\mu = E(V) = n_1 + \sum_{i=1}^{I} \frac{e^{\sum_{j=1}^{i} \lambda_j (n_j - n_i)}}{\sum_{j=1}^{i} \lambda_j} \left(1 - e^{-\sum_{j=1}^{i} \lambda_j (n_{i+1} - n_i)}\right).^{*}$$

Therefore, for large values of t,

(25) $$\frac{E(R_{0.})}{t} \cong \mu^{-1} .$$

This expected value can be partitioned into the expected number of planned replacements per unit time and the expected number of opportunistic replacements per unit time. By the law of large numbers, the fraction of the total number of replacements of part 0 that are planned is for large t approximately equal to p, the probability that starting with a new part 0, this part will not be replaced in the interval [0, N]. This probability is equal to:

(26) $$p = e^{-\sum_{i=1}^{I} \lambda_i (N - n_i)} .$$

It follows that in the long run $\frac{E(R_{00})}{t}$, the expected number (per unit time) of planned replacements of part 0 is given by:

(27) $$\frac{E(R_{00})}{t} \cong p\mu^{-1} .^{**}$$

Similarly, in the long run

---

*See Ref. 2.

**For a more rigorous proof see Appendix.

$$\frac{1}{t} \sum_{i=1}^{I} E(R_{oi}) , \tag{28}$$

the expected number (per unit time) of opportunistic replacements of part 0 may be expressed as:

$$\frac{1}{t} \sum_{i=1}^{I} E(R_{oi}) = (1-p)\,\mu^{-1} = q\mu^{-1} . \tag{29}$$

The total number of opportunistic replacements can be partitioned into I subsets, where the $i^{\underline{th}}$ subset contains those opportunistic replacements of part 0 which have been provoked by a failure of part i. That is, the $i^{\underline{th}}$ subset contains the joint replacements of part 0 and part i. If a regime of opportunistic replacement has been established, it may be useful for planning purposes to calculate the expected number (per unit time) of each of the I distinct opportunistic replacements. The utility of these calculations would of course depend on the difference in skills and equipment required for a joint replacement of part 0 and part i as opposed to a joint replacement of part 0 and part j, i, j = 1, 2, ... I; $i \neq j$.

Starting with a new part 0, q_{oi}, the probability that part 0 will be replaced jointly with part i is given by:

$$q_{oi} = \lambda_i \sum_{I \geq j \geq i} \int_{n_j}^{n_{j+1}} e^{\sum_{K=1}^{j} \lambda_K (n_K - V)} \, dV \tag{30}$$

where $n_{I+1} = N$.

Summing this expression over $i = 1, 2, \ldots, I$ yields the probability that starting with a new part 0, an opportunistic replacement will eventually occur. That is, part 0 will be replaced jointly with one of the inspected parts. Symbolically,

$$q = \sum_{i=1}^{I} q_{0i} \, . \tag{31}$$

In the long run, $\frac{E(R_{oi})}{t}$, the expected number (per unit time) of opportunistic replacements of part 0 and part i is given by:

$$\frac{E(R_{oi})}{t} = q_{oi} \, \mu^{-1} \text{.}^{*} \tag{32}$$

As before, calculation of the expected number of replacements of part 0 (opportunistic or planned) in the finite interval, [0, t] and calculation of the corresponding probability distributions both require the inversion of unfamiliar La Place transforms.

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*See Appendix.

V. A NUMERICAL EXAMPLE

For illustrative purposes, several support requirements will be computed for the hypothetical missile discussed in a previous paper.[7] The ballistic missile was composed of one uninspected part, the rocket engines, and three parts which are continuously inspected: the nozzle control units, the guidance and control system, and the re-entry vehicle. All parts were assumed to fail exponentially with respective failure rates λ_0, λ_1, λ_2, and λ_3. The costs measured in equivalent missile downtime, of replacing each of the parts separately are K_0, K_1, K_2, and K_3, respectively. Similarly, the costs of replacing pairs of parts (opportunistic replacements) are $K_{ij}(i, j = 0, 1, 2, 3, i \neq j)$. The structure of the missile is such that the rocket engines and nozzle control units can only be replaced together, that is, $K_0 = K_1 = K_{01}$. Each of the inspected parts is replaced at failure. The opportunistic replacement policy for the uninspected part is characterized by the following decision rule:

(1) Replace rocket engines whenever nozzle control units fail,

(2) Replace rocket engines if guidance and control system fails when $n_2 \leq t \leq N$,

(3) Replace rocket engines if re-entry vehicle fails when $n_3 \leq t \leq N$,

(4) Replace rocket engines at $t = N$,

where t is the age (since last replacement) of the rocket engines.

The critical numbers n_2, n_3, and N are calculated to maximize missile readiness. These critical numbers were calculated for the following set of data:*

$K_0 = K_1 = K_{01}$ .... 74 days

K_2 .... 57 days

K_3 .... 8 days

K_{02} .... 81 days

K_{03} .... 76 days

λ_1 .... .0022 failures/day

λ_2 .... .0048 failures/day

λ_3 .... .0044 failures/day.

Figure 1 displays optimal values of the critical numbers as a function of the rocket engine failure rate. For example, when λ_0 = .01 failures/day the optimal values of the critical numbers are:

n_2 ... 16 days

n_3 ... 74 days

N ... 109 days.

The implementation of this opportunistic replacement policy gives rise to a stochastic process with two distinct sets of support requirements. Those in the first set are independent of the rocket engine failure rate. These include $\frac{E(R_2)}{t}$, the expected number of replacements of the guidance and control system per unit time,

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*These data were obtained from a table of random numbers. It was assumed that the downtime associated with any replacement action would lie between zero and one hundred days and also that the mean time to failure for each part would lie between zero and one thousand days.

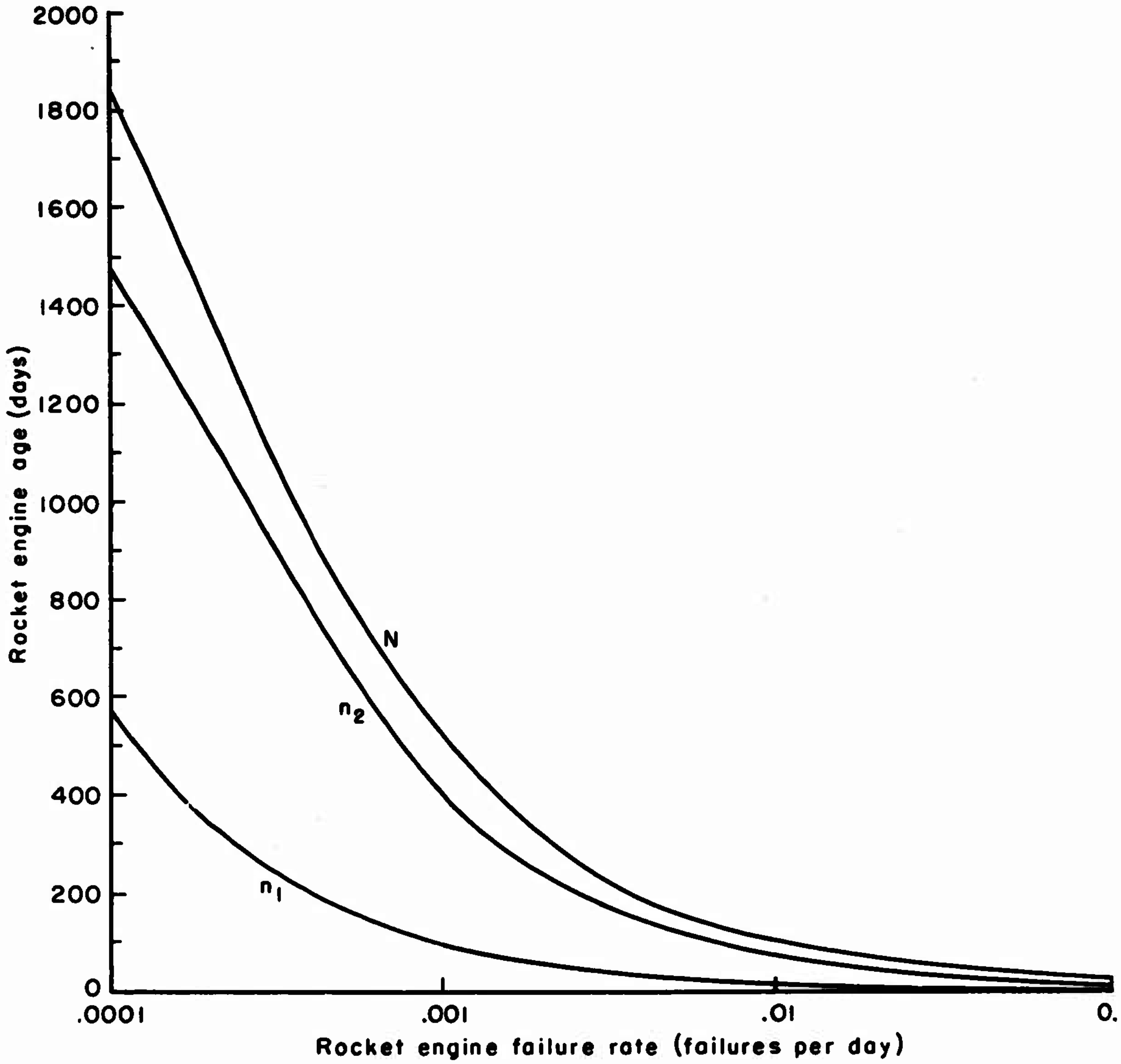

Fig. 1 — The critical numbers, n_2, n_3, and N, as a function of the rocket engine failure rate

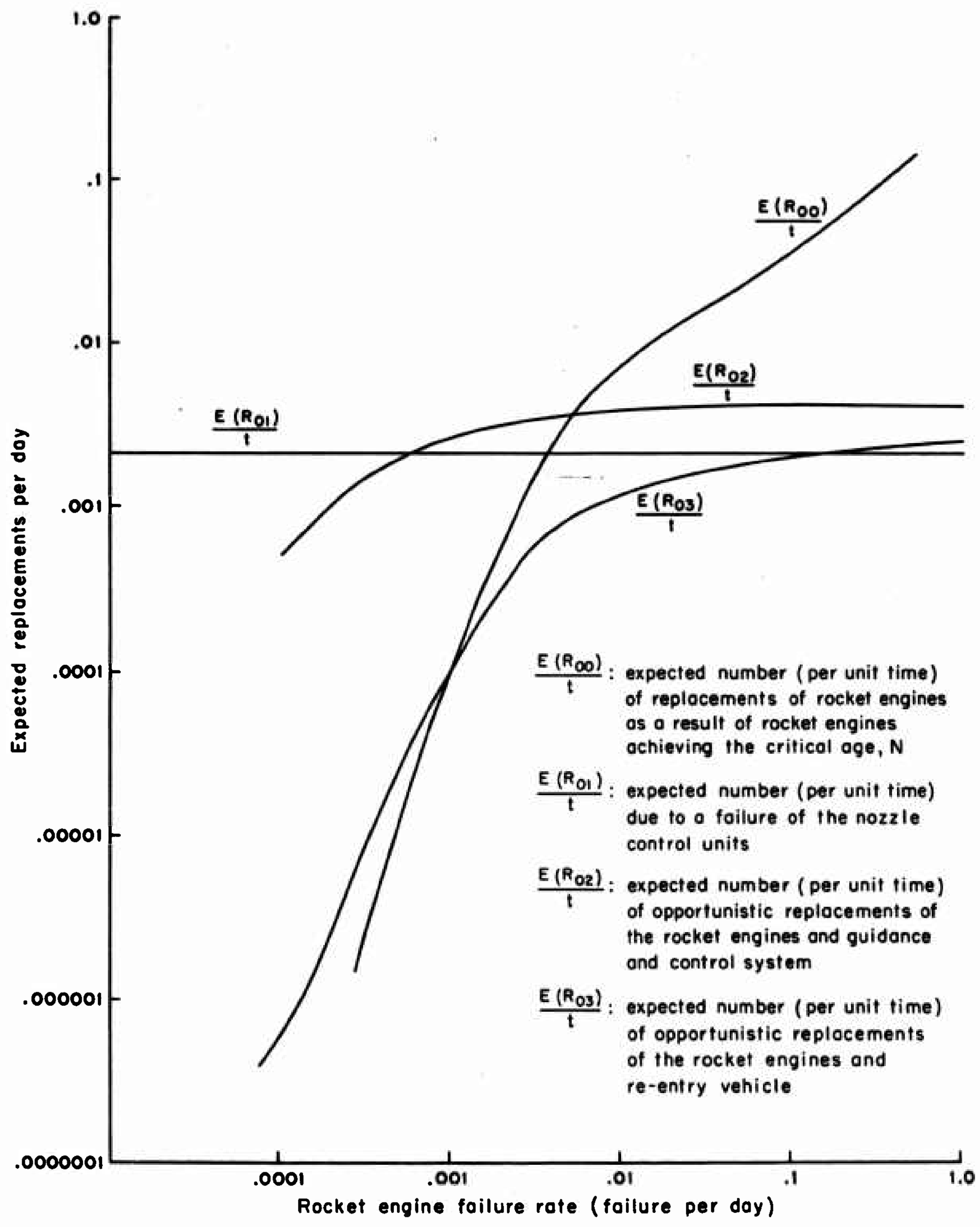

Fig. 2 — Several support requirements as a function of the rocket engine failure rate

$\frac{E(R_3)}{t}$, the expected number of replacements of the re-entry vehicle per unit time and $\frac{E(R_{01})}{t}$, the expected number of replacements of the rocket engines per unit time due to a failure of the nozzle control units. The values of these support requirements in this example are:

$\frac{E(R_2)}{t}$ .... .0048 replacements/day

$\frac{E(R_3)}{t}$ .... .0044 replacements/day

$\frac{E(R_{01})}{t}$ .... .0022 replacements/day

The support requirements in the second set are dependent on the rocket engine failure rate. These include $\frac{E(R_{02})}{t}$, the expected number of opportunistic replacements of the rocket engines and guidance and control system per unit time, $\frac{E(R_{03})}{t}$, the expected number of opportunistic replacements of the rocket engines and re-entry vehicle per unit time and $\frac{E(R_{00})}{t}$, the expected number (per unit time) of replacements of the rocket engines as a result of the rocket engines achieving the critical age, N.

Figure 2 illustrates the sensitivity of these support requirements to changes in the rocket engine failure rate. For example when the rocket engine failure rate increases from .001 failures per day to .01 failures per day the values of these support requirements increase from

$\frac{E(R_{02})}{t}$ .... .005 replacements/day

$\frac{E(R_{03})}{t}$ .... 6×10^{-7} replacements/day

to

$$\frac{E(R_{00})}{t} \;\ldots\ldots\; 2 \times 10^{-8} \text{ replacements/day}$$

$$\frac{E(R_{02})}{t} \;\ldots\ldots\; .0034 \text{ replacements/day}$$

$$\frac{E(R_{03})}{t} \;\ldots\ldots\; .001 \text{ replacements/day}$$

$$\frac{E(R_{00})}{t} \;\ldots\ldots\; .0054 \text{ replacements/day}$$

This indicates that both the expected number (per unit time) of opportunistic replacements of the rocket engines and re-entry vehicle and the expected number (per unit time) of replacements of the rocket engines due to mandatory replacement are highly sensitive to changes in the rocket engine failure rate. On the other hand, the expected number (per unit time) of opportunistic replacements of the rocket engines and the guidance and control system is relatively unaffected by changes in the engine failure rate.

VI. CONCLUSION

In this Memorandum several support requirements have been obtained for opportunistic replacement and inspection policies. Support requirements of this kind are quite important since they constitute the basic link between maintenance policies and inventory policies. Nevertheless as far as we know these support requirements have been relatively unexplored.

Whenever a regime of opportunistic maintenance is implemented, these measures should be useful for both inventory and maintenance management. The inventory manager can use this information to improve his stockage policies. The maintenance manager can also turn to these support requirements for guidance in his efforts to predict the relative frequencies of various maintenance actions. Procurement of specialized equipment and maintenance skills will of course be conditioned by his estimates of these relative frequencies.

APPENDIX

Equations (27) and (32) can be obtained by examining the subsidiary renewal process $\{U_{ij}\}$, $i = 1, 2, \ldots, I$; $j = 1, 2, \ldots,$ where

$$U_{ij} = \begin{cases} 1, & \text{if the } j^{\underline{th}} \text{ replacement of part 0} \\ & \text{is done jointly with part } i. \\ 0, & \text{otherwise.} \end{cases}$$

The expected value of this random variable is simply the probability of an opportunistic replacement of part 0 and part i, that is,

(1) $$E(U_{ij}) = q_{0i}\ .^{*}$$

Let $E[R_{0i}(t)]$ and $E[R_{0.}(t)]$ be, respectively, the expected number of opportunistic replacements of part 0 and part i in the interval [0, t] and the expected number of replacements (opportunistic or planned) in the interval [0, t]. The expected value, $E[R_{0i}(t)]$, is bounded by:

(2) $$E(U_{ij})(E[R_{0.}(t)]+1) - 1 \leq E[R_{0i}(t)] \leq E(U_{ij})(E[R_{0.}(t)] + 1)^{**}.$$

These bounds are a consequence of the following result[13] from sequential analysis,

(3) $$E(U_{i1}+U_{i2}+ \ldots +U_{i,R_{0.}(t)+1}) = E(U_{ij})(E[R_{0.}(t)]+1)\ .$$

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*See Eqn. (30).

**See p. 75 of Ref. 12.

We wish to show that

$$\lim_{t\to\infty} \frac{E[R_{0i}(t)]}{t} = q_{0i}\, \mu^{-1} \;{}^{*} \qquad (4)$$

By Eq. 2 we know that:

$$\frac{E(U_{ij})(E[R_{0.}(t)+1)-1}{t} \leq \frac{E[R_{0i}(t)]}{t} \leq \frac{E(U_{ij})(E[R_{0.}(t)]+1)}{t} \;, \qquad (5)$$

and by the elementary renewal theorem.[11]

$$\lim_{t\to\infty} \frac{E[R_{0.}(t)]}{t} = \mu^{-1} \;,$$

from which Eq. 4 follows immediately.

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*See Eq. 24.

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